Solution by H. C. WHITAKER. C. E., Ph. D., Manual Training School, Philadelphia, Pa.; MARCUS BAKER, U. S. Geological Survey, Washington, D. C.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

Plotting the curves represented by the two equations, they are seen to intersect at (0, 1) and also in the neighborhood of (.6, .8).

$$\frac{\log(7-6^y)}{\log 5}-\frac{\log(4-3^y)}{\log 2}=0.$$

By Double Position the roots near .6, .8 are found to be $x=.565585$, $y=.841307$; these with the values $x=0$, $y=1$ seem to be the only real roots.

Also solved by *JOHN G. KELLER*, State Normal School, Albany, N. Y.
The Proposer sent results only, to-wit: $x=.56556312$, $y=.8413092$.

GEOMETRY.

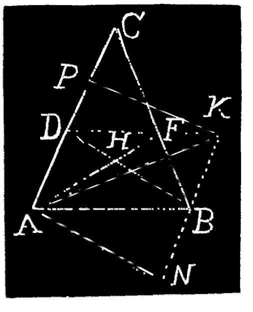
123, Proposed by P. C. CULLEN, Indianola, Iowa.

If the bisectors of the base angles of a triangle are equal the triangle is isosceles.

I. Direct Demonstration by G. I. HOPKINS, A. M., Professor of Physics and Astronomy, High School, Manchester, N. H.

CONSTRUCTION. Make $\angle BDK=\angle FAB$, and draw through B the line NK, making $\angle DBK=\angle AFB$. Draw the perpendiculars KP and AN, and join A and K.

DEMONSTRATION. Since angles FAB and AFB are two angles of a triangle, DK and NK will meet.



$\therefore$ triangles DBK and AFB are equal.

$\therefore$ $DK=AB$ and $BK=BF$.

$\angle AHB=\angle ADH+\angle DAH=\angle ADH+\angle BDK$.

$\therefore$ $\angle AHB=\angle ADK$. Also $\angle AHB=\angle HBF+\angle HFB=\angle HBA+\angle HBK$.

$\therefore$ $\angle AHB=\angle ABK$. $\therefore$ $\angle ADK=\angle ABK$.

$\therefore$ $\angle KDP=\angle ABN$. $\therefore$ $\triangle DPK=\triangle ABN$.

$\therefore$ $AN=KP$, and $NB=DP$.

$\therefore$ $\triangle APK=\triangle AKN$.

$\therefore$ $AP=NK$, and $\therefore$ $AD=BK$. $\therefore$ $AB=FB$. $\therefore$ $\triangle ADK=\triangle AFB$.

$\therefore$ $\angle ABD=\angle FAB$. $\therefore$ $\angle DAB=\angle ABF$.

$\therefore$ $AC=BC$, and the triangle ABC is isosceles. Q. E. D.

II. Demonstration by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, University of Chicago, Chicago, Ill.

Let d_a be the length of the bisector of the angle A opposite to side a. Expressing the area of the given triangle and the areas of the triangles into which it is divided by the bisector of angle A, we get

$$\tfrac{1}{2}bc\sin A=\tfrac{1}{2}bd_a\sin\tfrac{1}{2}A+\tfrac{1}{2}cd_a\sin\tfrac{1}{2}A.$$

$$\therefore\ d_a=\frac{2bc}{b+c}\cos\tfrac{1}{2}A=\frac{2\sqrt{[s(s-a)bc]}}{b+c},\text{ where } 2s=a+b+c.$$

Hence, if $d_a=d_b$, $\dfrac{2\sqrt{[s(s-a)bc]}}{b+c}=\dfrac{2\sqrt{[s(s-b)ac]}}{a+c}$.

$$\therefore\ 2(a+c)^2(s-a)b=2(b+c)^2(s-b)a.$$

$$\therefore\ (a+c)^2(b+c-a)b-(b+c)^2(a+c-b)a=0.$$

$$\therefore\ (a-b)[c^3+c^2(a+b)+3abc+ab(a+b)]=0.$$

Since the second factor is positive, it does not vanish. Hence $a=b$.

We have finally received an elementary direct proof of this theorem. The proof by Professor Hopkins is, I believe, without a flaw, and is the proof so long sought for by a number of mathematicians, among whom was Isaac Todhunter. This demonstration of Professor Hopkins' was examined by one of the ablest mathematicians in this country, and was pronounced by him to be correct. The trigonometric proof by Dr. Dickson is also flawless. We are glad to publish both of these proofs since the demonstration which we gave in Vol. VII, page 223, has been assailed.

Last January a year ago, we received an indirect proof from Prof. A. Anderson, of the University of North Carolina. Professor Anderson's demonstration is free from error and is substantially the proof given in Dr. Halsted's Synthetic Geometry, page 44, though Professor Anderson's figure is drawn quite differently from the one in Dr. Halsted's Geometry.

We shall be pleased to receive, from our readers, opinions on the above demonstrations.

153. Proposed by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

If P, P', Q, Q' be the extremities of two chords of a conic section, and both chords pass through the point A, show that the sum of the squares of the reciprocals of AP, AP', AQ, AQ' is constant.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics in The Temple College, Philadelphia, Pa.

$$\frac{1}{AP}=\frac{1+e\cos\theta}{l},\quad \frac{1}{AP'}=\frac{1+e\cos(\pi+\theta)}{l}=\frac{1-e\cos\theta}{l}.$$

$$\frac{1}{AQ}=\frac{1+e\cos(\frac{1}{2}\pi+\theta)}{l}=\frac{1-e\sin\theta}{l}.$$

$$\frac{1}{AQ'}=\frac{1+e\cos(3\pi/2+\theta)}{l}=\frac{1+e\sin\theta}{l}.$$

$$\therefore\frac{1}{(AP)^2}+\frac{1}{(AP')^2}+\frac{1}{(AQ)^2}+\frac{1}{(AQ')^2}$$

$$=\frac{(1+e\cos\theta)^2+(1-e\cos\theta)^2+(1-e\sin\theta)^2+(1+e\sin\theta)^2}{l^2}=\frac{2(2+e^2)}{l^2}=\text{a constant}.$$

158. Proposed by JOHN MACNIE, A. M., Professor of Latin, University of North Dakota.

Show by a simple diagram that:

(*a*) If the angle-sum of an equilateral triangle is constant. that constant is a straight angle.